American University of Beirut
Math 102
Quiz I (Spring 2010)

Time 50 minutes

Name: ____________________ **ID#:** ____________________

Circle your problem solving section number below:

Section 5 8:00 Tu **Section 6** 9:30 Tu **Section 7** 12:30 Tu **Section 8** 2:00 Tu

Solution

Question	Grade
1, 2	/ 18
3	/ 20
4	/ 62
Total	/ 100

1- Solve for x

(4 pts) $$3^{\log_3 (x^2)} = 5e^{\ln x} - 3\left[10^{\log_{10}(2)}\right].$$

$x^2 = 5x - 6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

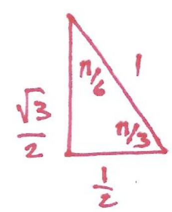
$x = 2$, or $x = 3$

2- Determine

(6 pts) **a.** $\cos\left(\sin^{-1}\left(\frac{-1}{2}\right)\right)$

$\sin^{-1}\left(\frac{-1}{2}\right) = x \qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$-\frac{1}{2} = \sin x \Rightarrow x = -\frac{\pi}{6}$



$\cos\left(\frac{-\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

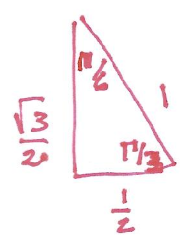
$\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \frac{\sqrt{3}}{2}$

(8 pts) **b.** $\sin\left(\cos^{-1}\left(\frac{-1}{2}\right)\right)$

$\cos^{-1}\left(-\frac{1}{2}\right) = x \qquad x \in [0, \pi]$

$\frac{-1}{2} = \cos x \Rightarrow x = \pi - \frac{\pi}{3}$

$x = \frac{2\pi}{3}$



$\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = \frac{\sqrt{3}}{2}$

3- Find the derivative $\frac{dy}{dx}$:

(4 pts) $y = \ln(\ln x)$

$\frac{dy}{dx} = \frac{1}{\ln x}(\ln x)' = \frac{1}{x \ln x}$

(4 pts) $y = \tan^{-1}(e^{3x}) + 3^x$

$y' = \frac{1}{1+(e^{3x})^2} \cdot 3e^{3x} + 3^x \ln 3$

(6 pts) $y = (x^2+3)^x$

$y = (x^2+3)^x$

$\ln y = \ln (x^2+3)^x$

$\ln y = x \ln(x^2+3)$

$\frac{y'}{y} = \ln(x^2+3) + x \frac{1}{x^2+3} \cdot 2x$

$y' = \left[\ln(x^2+3) + \frac{2x^2}{x^2+3}\right] y$

(6 pts) $y = x^{\ln x}$

$\ln y = \ln(x^{\ln x}) = \ln x \ln x = (\ln x)^2$

$\frac{y'}{y} = 2\ln x \cdot (\ln x)' = \frac{2\ln x}{x}$

$y' = 2\frac{\ln x}{x} \cdot y$

4- Evaluate the following integral

(5 pts) $\int_0^2 e^{\ln x}\,dx = \int_0^2 x\,dx = \left.\frac{x^2}{2}\right]_0^2 = \frac{4}{2} - 0 = 2$

(5 pts) $\int \frac{dy}{1-\cos y} = \int \frac{1+\cos y}{(1+\cos y)(1-\cos y)}\,dy = \int \frac{1+\cos y}{1-\cos^2 y}\,dy$

$= \int \frac{1+\cos y}{\sin^2 y}\,dy = \int \frac{dy}{\sin^2 y} + \int \frac{\cos y}{\sin^2 y}\,dy$

$= -\cot y - [\sin y]^{-1} + C$

(5 pts) $\int_0^{\pi} \sqrt{(1-\cos 2x)}\;dx = \int_0^{\pi} \sqrt{2\sin^2 x}\;dx = \sqrt{2}\int_0^{\pi} \sqrt{\sin^2 x}\,dx$

$= \sqrt{2}\int_0^{\pi} |\sin x|\,dx = \sqrt{2}\int_0^{\pi} \sin x\,dx = \sqrt{2}\left[-\cos x\right]_0^{\pi}$

$= -\sqrt{2}\,[-1-1] = 2\sqrt{2}$

(5 pts) $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}\,dx$

let $u = \sin^{-1} x$

$du = \frac{dx}{\sqrt{1-x^2}}$

$= \int e^u\,du = e^u + C = e^{\sin^{-1} x} + C$

(5 pts) $\int x^3 \sin 2x \, dx$

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	$+$	$\sin 2x$
$3x^2$	$-$	$-\frac{1}{2}\cos 2x$
$6x$	$+$	$-\frac{1}{4}\sin 2x$
6	$-$	$+\frac{1}{8}\cos 2x$
0		$\frac{1}{16}\sin 2x$

$$-\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2 \sin 2x + \frac{6x}{8}\cos 2x - \frac{6}{16}\sin 2x + C$$

(5 pts) $\int \ln x \, dx$

let $\ln x = u$ $\quad dx = dv$

$\frac{dx}{x} = du$ $\quad x = v$

$$= x\ln x - \int \frac{x\,dx}{x} = x\ln x - x + C$$

(5 pts) $\int \frac{x}{x^2+3x-4} \, dx$

$$\frac{x}{x^2+3x-4} = \frac{x}{(x-1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+4}$$

$$x = A(x+4) + B(x-1)$$

$x=1 \Rightarrow 5A = 1 \Rightarrow A = 1/5$

$x=-4 \Rightarrow -4 = -5B \Rightarrow B = \frac{4}{5}$

$$= \int \frac{1/5}{x-1}\,dx + \int \frac{4/5}{x+4} = \frac{1}{5}\int \frac{dx}{x-1} + \frac{4}{5}\int \frac{dx}{x+4}$$

$$= \frac{1}{5}\ln|x-1| + \frac{4}{5}\ln|x+4| + C$$

(8 pts) $\displaystyle\int \frac{x^2+2}{x^3-x^2+x-1}\,dx = \int \frac{x^2+2}{x^2(x-1)+(x-1)}\,dx = \int \frac{x^2+2}{(x^2+1)(x-1)}\,dx$

$$\frac{x^2+2}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$$

$x^2+2 = A(x^2+1) + (Bx+C)(x-1)$

$x^2+2 = Ax^2 + A + Bx^2 - Bx + Cx - C$

$x^2+2 = (A+B)x^2 + (C-B)x + A - C \Rightarrow \begin{cases} A+B=1 \\ C-B=0 \rightarrow C=B \\ A-C=2 \end{cases}$

$\begin{cases} A+B=1 \\ A-B=2 \end{cases}$ $A = 3/2$ $B = 1-A = 1-3/2 = -\frac{1}{2}$ $C = -\frac{1}{2}$

$= \frac{3}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{x+1}{x^2+1}\,dx = \frac{3}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{x\,dx}{x^2+1} - \frac{1}{2}\int \frac{dx}{x^2+1}$

$= \frac{3}{2}\ln|x-1| - \frac{1}{4}\ln|x^2+1| - \frac{1}{2}\tan^{-1}x + C$

(8 pts) $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-1}}$

let $e^x = u$ $e^x dx = du$ $dx = \frac{1}{e^x}du = \frac{1}{u}du$

$= \int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}u + C = \sec^{-1}(e^x) + C$

(8 pts) $\displaystyle\int \left(1+\frac{1}{x}\right)\tan\left(2x+\ln x^2\right)dx$

Let $2x + \ln x^2 = u$

$2x + 2\ln x = u$

$2(x+\ln x) = u \Rightarrow 2\left(1+\frac{1}{x}\right)dx = du \Rightarrow \left(1+\frac{1}{x}\right)dx = \frac{1}{2}du$

$= \frac{1}{2}\int \tan u\,du = -\frac{1}{2}\ln|\cos u| + C = -\frac{1}{2}\ln|\cos(2x+\ln x^2)| + C$